Pricing Fundamentals

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1 Self-financing portfolio

A portfolio $V(t)$ is self-financing if its value is given by

$$V(t) = \sum_{1 \leqslant i \leqslant N} w_i(t) S_i(t)$$

where w_i are the weights and S_i are the assets.

2 No-arbitrage

An arbitrage opportunity arises when one can construct a self-financing portfolio such that:

- $V(0) = 0$
- $P(V(T) \geq 0) = 1$ for maturity T
- $P(V(T) > 0) > 0$ for maturity T

3 Complete markets

A market is complete if all the square integrable random variables can be obtained as a terminal value of a self-financing portfolio i.e.

$$\forall X \in \{Y : Y \equiv (\Omega, \mathcal{F}_t, P), E(X^2) < \infty\}, \exists V \text{ s.t. } X = V(T)$$

4 Equivalent martingale measure

A probability measure Q with numeraire N_t is called equivalent martingale measure to P if:

- $\exists \{D_{PQ}(\omega) > 0, D_{QP}(\omega) >\}$ such that

  $$dP(\omega) = D_{PQ}(\omega) dQ(\omega)$$

  and,

  $$dQ(\omega) = D_{QP}(\omega) dP(\omega)$$

- The relative process $S_i^N(t)$ is a martingale i.e.

  $$S_i^N(s) = E\left[S_i^N(t) | \mathcal{F}_s\right], \text{ where } S_i^N(t) = \frac{S_i(t)}{N(t)}$$

5 Arbitrage-free pricing

Theorem 1: A market is arbitrage free iff there exists an equivalent martingale measure Q. This means that there exists a numeraire and measure under which the relative price process is a martingale.

Theorem 2: An arbitrage free market is complete iff the equivalent martingale measure Q is unique.